

Refined Analysis of the Electroweak Precision Data[†]

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Abstract

We refine our recent analysis of the electroweak precision data at the Z^0 pole by including the hadronic decay modes of the Z^0 . Within the framework of an effective Lagrangian we parametrize $SU(2)$ violation by the additional process-specific parameters Δy_ν , Δy_h , and Δy_b (for the $Z^0\nu\bar{\nu}$, $Z^0q\bar{q}$, and $Z^0b\bar{b}$ vertices) together with the previously introduced parameters Δx , Δy , and ε . We find that a six-parameter analysis of the experimental data is indeed feasible, and it is carried out in addition to a four-parameter fit for Δx , Δy , ε , and Δy_b only. We reemphasize that the experimental data have become sensitive to the (combined) magnitude of the vertex corrections at the $W^+l\bar{\nu}$ ($W^-l\nu$) and $Z^0l\bar{l}$ vertices, Δy , which is insensitive to the notion of the Higgs mechanism but dependent on the non-Abelian, trilinear vector-boson coupling. Full explicit analytical results for the standard one-loop predictions for the above-mentioned parameters are given, and the leading two-loop top-quark effects are included. The analytic formulae for the analysis of the experimental data in terms of the parameters Δx , Δy etc. are presented in order to encourage experimentalists to pursue such an analysis by themselves with future data.

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1 Introduction

Based on an effective Lagrangian [1] which parameterizes $SU(2)$ violation in terms of three parameters, $\Delta x, \Delta y, \varepsilon$, we have recently presented an analysis [2] of the experimental data [3] on the leptonic width, Γ_l , of the Z^0 , the leptonic mixing angle, \bar{s}_W^2 , and the W^\pm mass, M_{W^\pm} . In systematically discriminating fermion-loop corrections to the W^\pm , γ , Z propagators from all the other one-loop corrections which depend on the empirically unknown non-Abelian couplings and the properties of the Higgs scalar, we obtained the striking result that contributions beyond the pure fermion-loop corrections are required for consistency with the experimental data. This is a consequence of the high accuracy reached by the most recent data. More specifically, we found that such bosonic contributions are required in the parameter Δy , which connects the W^\pm -fermion-coupling (determined via μ^\pm decay) with the W^0 -fermion-coupling at the Z^0 -mass shell, $g_{W^\pm}^2(0) \equiv 4\sqrt{2}G_\mu M_W^2 = (1+\Delta y)g_{W^0}^2(M_Z^2)$. Within the standard model, the bosonic contribution to Δy is practically independent of the Higgs mass, M_H , but contains vertex corrections to the couplings of the W^\pm and the Z^0 to fermions which depend on the trilinear vector-boson self-couplings: the experimental data have indeed become sensitive to bosonic loops, i.e. to corrections which involve the bosonic couplings of the vector bosons among each other.

In the present paper, we extend our previous results by including the hadronic observables, the total Z^0 width, Γ_T , as well as the partial widths for Z^0 decay into hadrons, Γ_h , and for Z^0 decay into $b\bar{b}$ pairs, Γ_b , in our analysis. Accordingly, we generalize our effective Lagrangian to include vertex modifications which are specific for the couplings of the Z^0 to the different quark flavors. This amounts to introducing the parameters $\Delta y_{u,d} = \Delta y_{c,s}$ and Δy_b (in addition to the vertex modification at the $Z^0 \nu\bar{\nu}$ vertex, Δy_ν). Explicit analytical expressions for these parameters in the standard electroweak theory are given.

In conjunction with the inclusion of the hadronic observables, we also update our previous analysis [2] by using the most recent experimental data [4].

In the analysis of the data we will proceed in two steps. In a first step, we present the results of a purely phenomenological six-parameter analysis, determining the parameters Δy_ν , a linear combination of Δy_u and Δy_d to be called Δy_h , and Δy_b (in addition to the “leptonic” parameters $\Delta x, \Delta y, \varepsilon$) from the above-mentioned six observables, M_{W^\pm} , Γ_l , \bar{s}_W^2 and $\Gamma_T, \Gamma_h, \Gamma_b$. It is remarkable that such a six-parameter analysis of the data is possible with reasonable experimental errors. In a second step, we take advantage of the fact that Δy_ν and Δy_h can actually be reliably calculated in the standard model solely from the empirically well-known couplings of the vector-bosons to fermions. The parameters Δy_ν and Δy_h are indeed on the same footing as the contributions $\Delta x^{\text{ferm}}, \Delta y^{\text{ferm}}, \varepsilon^{\text{ferm}}$ to $\Delta x, \Delta y, \varepsilon$ resulting from the fermion-loop corrections to the vector-boson propagators already employed previously. The number of free parameters thus being reduced to four, we will present fits to the mentioned six observables as well as fits to five observables, by singling out and excluding Γ_b .

A few general comments on our procedure seem appropriate:

- It is sometimes argued that a direct comparison of the observables with (standard) theoretical predictions may be sufficient, rather than comparing with the parameters of an effective Lagrangian essentially introduced by $SU(2)$ -symmetry requirements. This is by no means true, however. Indeed, while the necessity for bosonic corrections

in addition to fermion-loop propagator corrections became apparent in our direct comparison with the observables M_{W^\pm} , \bar{s}_W^2 , Γ_1 (see Figs. 1–3 in Ref. [2]), the detailed nature of these corrections only became clear *after the transition to the parameters* $\Delta x, \Delta y, \varepsilon$. More precisely, it turned out that approximating Δx and ε by fermion loops only, $\Delta x \approx \Delta x^{\text{ferm}}$, $\varepsilon \approx \varepsilon^{\text{ferm}}$, lead to agreement with experiment. On the other hand, a non-zero contribution to Δy in addition to the fermion loops, Δy^{ferm} , was shown to be necessary. In the standard theory it is provided by the above-mentioned contribution to Δy which corresponds to (combined) vertex corrections at the Z^0 and the W^\pm couplings to fermions and depends on the non-Abelian structure of the theory. However, it turned out that the experimental resolution of Δy does not yield any constraint with respect to the Higgs mechanism. In fact, Δy can be predicted in an electroweak massive vector-boson theory [5].

- As in our previous work [2], in clearly separating fermion-loop effects to γ *as well as* Z^0 and W^\pm propagation (and additional effects in the present paper which are on the same ground theoretically), we clearly differ from related work [6] in which the $\alpha(M_Z^2)$ -Born approximation (i.e. fermion loops in γ -*propagation only*) is compared with the data and the full electroweak theory. While such an analysis shows that contributions beyond the $\alpha(M_Z^2)$ -Born approximation are needed, it does not provide information about the detailed nature of these additional contributions within electroweak theory. With respect to the experimental data on the effective mixing angle, \bar{s}_W^2 , the effect of separating bosonic and fermionic loops was also explored in Ref. [7]. The measured value of the W^\pm mass was focussed on in Ref. [8] with respect to the necessity of corrections beyond the $\alpha(M_Z^2)$ -Born approximation.
- We note that our work differs from Refs. [9, 10] in so far as our emphasis is on precision tests of the bosonic sector of the electroweak theory via fully separating fermionic and bosonic corrections, while the main emphasis in Refs. [9, 10] is put on testing extensions of the standard theory, such as supersymmetry or technicolor.

In Sect. 2, we present the extended version of our effective Lagrangian for the description of Z^0 interactions at one-loop level. In Sect. 3, we turn to the analytic results for the parameters introduced in addition to the ones used previously, referring to the appendix for a few technical details. In Sect. 4, we briefly summarize (known) two-loop and QCD corrections relevant for the analysis of the observables in Sect. 5. Final conclusions are drawn in Sect. 6.

2 The effective Lagrangian

2.1 The leptonic sector

We first of all consider the case of charged leptons¹ interacting with the charged and neutral vector bosons W^\pm , γ and Z^0 . We start by slightly refining the treatment [1, 2]

¹Lepton universality is assumed throughout in our effective Lagrangian.

of neutral-current interactions in the leptonic sector in terms of an effective Lagrangian. Starting from the charged-current interaction in

$$\mathcal{L}_C = -\frac{1}{2}W^{+\mu\nu}W_{\mu\nu}^- - \frac{g_{W^\pm}}{\sqrt{2}}(j_\mu^+W^{+\mu} + h.c.) + M_{W^\pm}^2 W_\mu^+ W^{-\mu}, \quad (1)$$

$SU(2)$ symmetry is broken in the transition to neutral-current interactions by introducing the parameters x and y via

$$M_{W^\pm}^2 = xM_{W^0}^2 = (1 + \Delta x)M_{W^0}^2, \quad (2)$$

and

$$4\sqrt{2}G_\mu M_{W^\pm}^2 \equiv g_{W^\pm}^2(0) = yg_{W^0}^2(M_Z^2) = (1 + \Delta y)g_{W^0}^2(M_Z^2), \quad (3)$$

as well as mixing of strength λ between the electromagnetic and neutral W^0 fields,

$$\mathcal{L}_{\text{mix}} = -\frac{\lambda}{2}A_{\mu\nu}W^{0,\mu\nu}. \quad (4)$$

As in Ref. [1, 2], the parameter λ will be replaced by ε via²

$$\lambda \equiv \frac{e(M_Z^2)}{g_{W^0}(M_Z^2)}(1 - \varepsilon), \quad (5)$$

where g_{W^0} refers to the coupling of a charged lepton to the neutral member of the (W^\pm, W^0) -triplet. For definiteness, we have indicated the energy scales as arguments of the various couplings. The electromagnetic coupling at the Z^0 mass has been denoted by $e(M_Z^2)$ with

$$\frac{e^2(M_Z^2)}{4\pi} = \alpha(M_Z^2) \approx 1/129, \quad (6)$$

where $\alpha(M_Z^2)$ includes the “running” of electromagnetic vacuum polarization induced by the light fermions [12].

A few additional remarks on the parameters introduced in (2) to (5) are appropriate and useful in connection with the realization of non-zero values of these parameters by loop corrections to be discussed in Sect. 3. As x in (2) is defined as a mass ratio, it is obviously a “universal” quantity, i.e., it is independent of the external particles participating in a specific process described by Lagrangian (1) and its neutral-current counterpart to be defined below. Likewise, the parameter λ in (5), as a mixing strength among neutral boson fields, is a universal quantity. In contrast, as y in (3) relates the charged current coupling deduced from the specific process of μ^\pm decay to the coupling of the neutral component of the (W^\pm, W^0) -triplet to charged leptons, the parameter y obviously contains a process-specific contribution. Assuming that y is induced by loop corrections, as in the standard theory to be discussed in Sect. 3, y will also contain a process-independent, i.e. universal part, as according to (3) it relates quantities at different energy scales. Accordingly, in linear expansion (3) may be written as

$$g_{W^\pm}(0) = g_{W^0}(M_Z^2) \left(1 + \frac{1}{2}\Delta y^{\text{ZPD}} + \frac{1}{2}\Delta y^{\text{WPD}} + \frac{1}{2}\Delta y^{\text{univ}} \right), \quad (7)$$

²Our notation is an outgrowth of several steps which lead to the underlying Lagrangian. The mixing parameter ε was first introduced in Ref. [11], and Δx , Δy in Ref. [1].

where Δy^{ZPD} and Δy^{WPD} refer to the Z^0 and W^\pm vertices, respectively, and Δy^{univ} denotes an additional universal part. As the expression (5) for the universal (neutral-current) mixing strength, λ , in terms of ε contains the process-specific coupling $g_{W^0}^2(M_Z^2)$, also ε in (5) has to contain a process-specific contribution,

$$\lambda = \frac{e(M_Z^2)}{g_{W^0}(M_Z^2)} (1 - \varepsilon^{\text{ZPD}} - \varepsilon^{\text{univ}}). \quad (8)$$

which must be identical to the Z^0 -process-specific part in $g_{W^0}^2(M_Z^2)$ in (7), i.e.,

$$\varepsilon^{\text{ZPD}} = \frac{1}{2} \Delta y^{\text{ZPD}}. \quad (9)$$

This relation will be seen to emerge also in Sect. 3, where the expressions for $\Delta x, \Delta y, \varepsilon$ induced by loop effects in the $SU(2) \times U(1)$ theory will be given.

In order to construct the effective Z^0 interaction, we still have to specify the interaction of the photon and the interaction of the third component of the (W^\pm, W^0) -triplet with fermions. Noting that the effective Lagrangian will have to contain standard one-loop interactions as a special case, we couple the photon (at the Z^0 -mass scale) to a parity violating current via the replacement

$$A_\mu j_{\text{em}}^\mu \rightarrow A_\mu Q (\bar{\psi}_L \gamma^\mu \psi_L + (1 + \delta) \bar{\psi}_R \gamma^\mu \psi_R), \quad (10)$$

where obviously δ has to vanish in the limit of real photons interacting with fermions.

Skipping an explicit presentation of the W^0 interaction [1, 2], upon diagonalization, the Z^0 part of the neutral-current Lagrangian becomes,

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{M_{W^0}^2}{2(1 - \lambda^2)} Z_\mu Z^\mu \\ & - \frac{g_{W^0}}{\sqrt{1 - \lambda^2}} \left[j_\mu^3 - \frac{e\lambda}{g_{W^0}} Q_1 (\bar{\psi}_L \gamma_\mu \psi_L + (1 + \delta) \bar{\psi}_R \gamma_\mu \psi_R) \right] Z^\mu, \end{aligned} \quad (11)$$

where $Q_1 = -1$ is the lepton charge. Introducing the following linear combinations as auxiliary quantities,

$$x' = x + 2s_0^2 \delta, \quad y' = y - 2s_0^2 \delta, \quad \varepsilon' = \varepsilon - \delta, \quad (12)$$

and keeping δ only in linear order (δ in, e.g., the standard electroweak theory turns out to be extremely small, $\delta \sim 10^{-4}$), the neutral-current Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{M_{W^\pm}^2}{2x' (1 - \bar{s}_W^2 (1 - \varepsilon'))} Z_\mu Z^\mu \\ & - \frac{g_{W^\pm}}{\sqrt{y' (1 - \bar{s}_W^2 (1 - \varepsilon'))}} [j_\mu^3 - \bar{s}_W^2 j_{\text{em}, \mu}] Z^\mu. \end{aligned} \quad (13)$$

The (leptonic) weak mixing angle, \bar{s}_W^2 , is empirically determined by the charged lepton asymmetry at the Z^0 resonance,

$$\bar{s}_W^2 = \frac{e^2(M_Z^2)}{g_{W^\pm}^2(0)} y' (1 - \varepsilon'), \quad (14)$$

and $g_{W^\pm} \equiv g_{W^\pm}(0)$ is given in (3). s_0^2 in (12) is defined via

$$s_0^2(1 - s_0^2) = s_0^2 c_0^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_\mu M_Z^2}. \quad (15)$$

We note that Lagrangian (13) has the same form as Lagrangian (10) in Ref. [2], apart from the replacement of x, y, ε by x', y', ε' . This replacement takes into account a non-vanishing value of δ , which is in fact present in the standard theory even though it is very small. Nevertheless, from a principal point of view, it has to be introduced in order to assure universality of x in (2). In the subsequent analysis, standard model values for δ will be inserted. This can be done without loss of generality for two reasons. First of all, departures in δ from its standard value according to (12) can always be absorbed in the parameters x, y, ε to be determined from the experimental data. Secondly, it will be seen that in the $SU(2) \times U(1)$ theory δ is on equal footing with the fermion-loop corrections and can be calculated equally reliably.

Finally, we express the weak mixing angle, \bar{s}_W^2 , the W^\pm mass, M_{W^\pm} , and the leptonic Z^0 width, Γ_1 , in terms of x', y', ε' ,

$$\begin{aligned} \bar{s}_W^2 (1 - \bar{s}_W^2) &= \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_\mu M_Z^2} \frac{y'}{x'} (1 - \varepsilon') \frac{1}{\left(1 + \frac{\bar{s}_W^2}{1 - \bar{s}_W^2} \varepsilon'\right)}, \\ \frac{M_{W^\pm}^2}{M_Z^2} &= (1 - \bar{s}_W^2) x' \left(1 + \frac{\bar{s}_W^2}{1 - \bar{s}_W^2} \varepsilon'\right), \\ \Gamma_1 &= \frac{G_\mu M_Z^3}{24\pi\sqrt{2}} \left[1 + (1 - 4\bar{s}_W^2)^2\right] \frac{x'}{y'} \left(1 + \frac{3\alpha}{4\pi}\right). \end{aligned} \quad (16)$$

Linearizing these observables also in $\Delta x, \Delta y, \varepsilon$ yields

$$\begin{aligned} \bar{s}_W^2 &= s_0^2 \left[1 - \frac{1}{c_0^2 - s_0^2} \varepsilon - \frac{c_0^2}{c_0^2 - s_0^2} (\Delta x - \Delta y) + (c_0^2 - s_0^2) \delta\right], \\ \frac{M_{W^\pm}}{M_Z} &= c_0 \left[1 + \frac{s_0^2}{c_0^2 - s_0^2} \varepsilon + \frac{c_0^2}{2(c_0^2 - s_0^2)} \Delta x - \frac{s_0^2}{2(c_0^2 - s_0^2)} \Delta y\right], \\ \Gamma_1 &= \Gamma_1^{(0)} \left[1 + \frac{8s_0^2}{1 + (1 - 4s_0^2)^2} \left\{ \frac{1 - 4s_0^2}{c_0^2 - s_0^2} \varepsilon + \frac{c_0^2 - s_0^2 - 4s_0^4}{4s_0^2(c_0^2 - s_0^2)} (\Delta x - \Delta y) + 2s_0^2 \delta \right\}\right], \end{aligned} \quad (17)$$

with

$$\Gamma_1^{(0)} = \frac{\alpha(M_Z^2)M_Z}{48s_0^2c_0^2} \left[1 + (1 - 4s_0^2)^2\right] \left(1 + \frac{3\alpha}{4\pi}\right). \quad (18)$$

Obviously, upon absorbing δ in x, y, ε according to (12), the relations (17) agree with relations (14) of Ref. [2] apart from the replacement of x, y, ε by x', y', ε' .

2.2 Generalization to arbitrary fermions

We turn to the generalization of the effective Lagrangian to the case of neutrinos and, in particular, quarks. Accordingly, for each flavor degree of freedom, we have to allow for a separate, process-specific vertex correction. We define

$$g_{W^\pm}^2(0) \equiv y_f g_{W^0, f}^2(M_Z^2) = (1 + \Delta y)(1 + \Delta y_f) g_{W^0, f}^2(M_Z^2), \quad (19)$$

where f stands for any one of $f = \nu, u, d, c, s, b$. Clearly, the case of $\Delta y_f \equiv 0$ corresponds to the charged-lepton case,

$$g_{W^0} \equiv g_{W^0, e, \mu, \tau}. \quad (20)$$

In addition, we allow for the process-specific correction in the electromagnetic current³ analogously to (10),

$$A_\mu j_{\text{em}}^\mu \rightarrow A_\mu Q_f \left(\bar{\psi}_L \gamma^\mu \psi_L + (1 + \delta + \delta_f) \bar{\psi}_R \gamma^\mu \psi_R \right). \quad (21)$$

Passing through the diagonalization procedure previously employed, we now have in generalization of (11)

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{M_{W^0}^2}{2(1 - \lambda^2)} Z_\mu Z^\mu \\ & - \frac{y_f g_{W^0}}{\sqrt{1 - \lambda^2}} \left[j_\mu^3 - \frac{e\lambda}{y_f g_{W^0}} Q_f \left(\bar{\psi}_L \gamma_\mu \psi_L + (1 + \delta + \delta_f) \bar{\psi}_R \gamma_\mu \psi_R \right) \right] Z^\mu. \end{aligned} \quad (22)$$

According to (22), for each fermion flavor, f , we have a specific coupling strength and a specific mixing angle. For $y_f = 1$, $\delta_f = 0$, we recover the lepton case (11). In terms of a more “physical” set of parameters the Lagrangian (22) takes the form

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{M_{W^\pm}^2}{2x' (1 - \bar{s}_W^2 (1 - \varepsilon'))} Z_\mu Z^\mu \\ & - \frac{g_{W^\pm} \left(1 + 2|Q_f| s_0^2 \delta_f + 2(|Q_f| - 1) s_0^2 \delta \right)}{\sqrt{y' y_f (1 - \bar{s}_W^2 (1 - \varepsilon'))}} \left[j_\mu^3 - \bar{s}_{W,f}^2 j_{\text{em}, \mu} \right] Z^\mu, \end{aligned} \quad (23)$$

where the weak mixing angle (for charged fermions) is given by

$$\bar{s}_{W,f}^2 = \sqrt{y_f} \bar{s}_W^2 + s_0^2 \left(1 - 2|Q_f| s_0^2 \right) \delta_f + 2(1 - |Q_f|) s_0^4 \delta. \quad (24)$$

The Lagrangian (23) generalizes (13) to the case of an arbitrary fermion f . The fact that δ_f in (23) cannot be absorbed by a redefinition of y_f in (23), (24) is of no practical relevance. Recall that the quark asymmetries depend on both the leptonic mixing angle, \bar{s}_W^2 , and the quark mixing angle, $\bar{s}_{W,q}^2$ ($f = q$). The dependence on $\bar{s}_{W,q}^2$, however, is extremely weak so that it is justified to replace $\bar{s}_{W,q}^2$ by \bar{s}_W^2 in the quark asymmetries. Consequently, δ_q cannot be separated from Δy_q by measuring the quark asymmetries. On the other hand, also δ_f can be reliably calculated in the standard model.

With the modified Lagrangian (23) we obtain for the $Z^0 \rightarrow \nu \bar{\nu}$ decay width, Γ_ν ,

$$\Gamma_\nu = \frac{G_\mu M_Z^3}{12\pi\sqrt{2}} \frac{x}{y y_\nu}, \quad (25)$$

³Strictly speaking, we would also have to allow for a coupling between the photon on the Z^0 -mass scale and the neutrino. However, in the Z^0 part of the neutral-current Lagrangian this effect would merely lead to a trivial redefinition of the parameter Δy_ν .

and for the $Z^0 \rightarrow q\bar{q}$ decay widths, Γ_q ,

$$\Gamma_q = \frac{G_\mu M_Z^3}{8\pi\sqrt{2}} \frac{x}{y y_q} \left[1 + \left(1 - 4|Q_q| \bar{s}_{W,q}^2 \right)^2 \right] \left(1 + 4|Q_q| s_0^2 (\delta + \delta_q) \right) \left(1 + Q_q^2 \frac{3\alpha}{4\pi} \right) R_{\text{QCD}}, \quad (26)$$

where δ and δ_q again are kept in linear order. Note that in analogy to the $\mathcal{O}(\alpha)$ QED factor we have also applied the QCD factor

$$R_{\text{QCD}} = 1 + \left(\frac{\alpha_s}{\pi} \right) + 1.41 \left(\frac{\alpha_s}{\pi} \right)^2 - 12.8 \left(\frac{\alpha_s}{\pi} \right)^3, \quad (27)$$

to the hadronic widths. R_{QCD} represents the one-, two- and three-loop QCD corrections [13] corresponding to massless quarks. The inclusion of finite-mass effects is described in Sect. 4. For completeness, we write down the linearized form of the observables in (24), (25) and (26) yielding

$$\Gamma_\nu = \Gamma_\nu^{(0)} [1 + \Delta x - \Delta y - \Delta y_\nu], \quad (28)$$

for Γ_ν and

$$\begin{aligned} \bar{s}_{W,q}^2 &= s_0^2 \left[1 - \frac{1}{c_0^2 - s_0^2} \varepsilon - \frac{c_0^2}{c_0^2 - s_0^2} (\Delta x - \Delta y) + \frac{1}{2} \Delta y_q + (1 - 2s_0^2 |Q_q|) (\delta + \delta_q) \right], \\ \Gamma_q &= \Gamma_q^{(0)} \left[1 + \frac{8s_0^2}{1 + (1 - 4|Q_q| s_0^2)^2} \left\{ \frac{|Q_q|(1 - 4|Q_q| s_0^2)}{c_0^2 - s_0^2} \varepsilon + \frac{2|Q_q| s_0^2 - 1}{4s_0^2} \Delta y_q \right. \right. \\ &\quad \left. \left. + \frac{c_0^2 - s_0^2 + 4|Q_q| s_0^4 (1 - 2|Q_q|)}{4s_0^2 (c_0^2 - s_0^2)} (\Delta x - \Delta y) + 2s_0^2 Q_q^2 (\delta + \delta_q) \right\} \right], \quad (29) \end{aligned}$$

for the hadronic observables. Here, we introduced the lowest-order decay widths

$$\begin{aligned} \Gamma_\nu^{(0)} &= \frac{\alpha(M_Z^2) M_Z}{24s_0^2 c_0^2}, \\ \Gamma_q^{(0)} &= \frac{\alpha(M_Z^2) M_Z}{16s_0^2 c_0^2} \left[1 + (1 - 4|Q_q| s_0^2)^2 \right] \left(1 + Q_q^2 \frac{3\alpha}{4\pi} \right) R_{\text{QCD}}. \quad (30) \end{aligned}$$

3 Analytic results in the standard model

In Ref. [2], it was our essential point to systematically distinguish between fermion-loop contributions *to γ as well as W^\pm and Z^0 propagation* (compare Fig. 1) and other contributions, which in general depend on the couplings of the vector bosons with each other.

As pointed out in the previous section, we first of all refine the treatment of Refs. [1, 2] by also separating the (numerically unimportant) contribution called δ in the previous section. This contribution leads to a discrimination between right-handed and left-handed fermions in the coupling to the photon (compare (10)) which in terms of Z^0 couplings corresponds to the diagrams shown in Fig. 2. These diagrams form a gauge-invariant set and only depend on the couplings of the bosons to fermions. Accordingly, they are basically of similar nature as the fermion loops in Fig. 1 and can be reliably calculated.

Figure 1: Fermion-loop contributions to gauge-boson propagators.

Figure 2: Feynman graphs relevant for the parameter δ .

The same conclusion will hold if the leptons in Fig. 10 are replaced by quarks, thus yielding δ_q . Both, δ and δ_q will be explicitly given below.

The second extension of our previous work is concerned with the vertex correction Δy for the case of quarks, Δy_q , and the neutrino, Δy_ν . Since Δy_q , Δy_ν are defined as deviations from the leptonic parameter, Δy , all universal and WPD corrections are already included in Δy so that Δy_q , Δy_ν are entirely furnished by ZPD contributions. It is important to distinguish between the cases of the light quarks $q = u, d, c, s$ on the one hand, and the case of the b-quark on the other hand. The diagrams relevant for the process-dependent contribution, Δy_b , are depicted in Fig. 3. Note that the diagrams involving the Yukawa coupling of the charged Goldstone scalar field, φ , to the fermion doublet (graphs Fig. 3c),f),i),k),l),m) for the (b,t)-doublet) do not contribute for light doublets. Moreover, one finds that the contribution of the diagram involving the non-Abelian vertex (i.e. the analogous ones to graph Fig. 3j) for $Z^0 \rightarrow b\bar{b}$) is already contained in Δy for light doublets since these diagrams coincide with the one in the lepton case (up to a necessary change in the sign if the charge flow in the loop is inverted). Consequently, Δy_ν and Δy_q for $q = u, d, c, s$ do not contain Yukawa and trilinear boson couplings, and accordingly are again *on the same footing as the fermion loops* in the gauge-boson propagators and can be reliably calculated. The situation is different for $Z^0 \rightarrow b\bar{b}$. In this case, indeed, we obtain a contribution due to Yukawa couplings and the trilinear non-Abelian vertex in conjunction with the dependence on the mass of the top-quark, m_t .

Figure 3: Feynman diagrams relevant for the parameter Δy_b .

We turn to a representation of the analytic results. Separating fermion loops (in γ, W^\pm, Z^0 propagation) and the remaining contributions which depend on the gauge-boson and the Higgs sector of the theory, as in Ref. [2], we have

$$a = a_{\text{ferm}} + a_{\text{bos}}, \quad \text{with} \quad a = \Delta x, \Delta y, \varepsilon, \quad (31)$$

and

$$a_{\text{bos}} = a_{\text{bos}}^{\text{univ}} + a^{\text{WPD}} + a^{\text{ZPD}}. \quad (32)$$

There is no reason to repeat the formulae given in Ref. [2] apart from the (minor, theoretically relevant, but numerically unimportant) change in the ZPD contributions due to the parameter δ introduced above.

It seems appropriate, however, to add a brief comment on the general structure of our results before explicitly quoting them. The (generally non-unique) splitting of the sum on the right-hand side in (32) into separately gauge-parameter-independent vacuum-polarization “univ” and vertex (“WPD”, “ZPD”) parts in Ref. [2] was carried out by employing a procedure known as pinch technique (see e.g. Ref. [14] and references therein). It yields a somewhat different splitting in the right-hand side in (32) than other methods [15, 16], and the results obtained for the vertex functions coincide with the ones of the “background field method” in the ’t Hooft-Feynman gauge [17].

The parameter Δx , defined by (2) is process-independent,

$$\Delta x_{\text{bos}}^{\text{univ}} \neq 0, \quad \Delta x^{\text{ZPD}} = \Delta x^{\text{WPD}} = 0. \quad (33)$$

The only independent process-dependent parameters are contained in the vertex modification Δy ,

$$\Delta y_{\text{bos}}^{\text{univ}} \neq 0, \quad \Delta y^{\text{ZPD}} \neq 0, \quad \Delta y^{\text{WPD}} \neq 0, \quad (34)$$

since the process-dependent part of ε , is related to Δy^{ZPD} via (compare (9))

$$\varepsilon_{\text{bos}}^{\text{univ}} \neq 0, \quad \varepsilon^{\text{ZPD}} = \frac{1}{2} \Delta y^{\text{ZPD}}, \quad \varepsilon^{\text{WPD}} = 0. \quad (35)$$

The explicit expressions for the universal and WPD contributions to $\Delta x, \Delta y, \varepsilon$ are exactly the ones given in Ref. [2]. The ZPD parts are different owing to the introduction of δ ,

$$\delta = -\frac{\alpha(M_Z^2)s_0^2}{8\pi c_0^2}(11 + 16C_1) = -\frac{\alpha(M_Z^2)s_0^2}{8\pi c_0^2}\left(11 - \frac{4}{3}\pi^2\right) = 0.20 \times 10^{-3}. \quad (36)$$

The constants $C_{1,2,3}$ and the function $f_1(x)$ are defined in App. B. The expression for ε^{ZPD} in the present notation is given by

$$\begin{aligned} \varepsilon^{\text{ZPD}} = \frac{\alpha(M_Z^2)}{4\pi s_0^2} & \left[(1 - 2s_0^2)^3 \frac{2C_1}{c_0^2} - (2 - s_0^2)^2 C_2 - 2c_0^4(3 - s_0^2)C_3 \right. \\ & \left. - \left(\frac{5}{2} - s_0^2\right) \left(\log(c_0^2) - 2c_0^2 f_1(c_0^2)\right) + \frac{17}{8c_0^2} - \frac{27s_0^2}{2c_0^2} + \frac{23s_0^4}{c_0^2} - \frac{13s_0^6}{c_0^2} \right], \end{aligned} \quad (37)$$

while Δx^{ZPD} vanishes according to (33), and Δy^{ZPD} follows from (35) upon inserting ε^{ZPD} from (37). We also note the numerical values of the process-specific parameters

$$\begin{aligned} \Delta y^{\text{ZPD}} &= 2\varepsilon^{\text{ZPD}} = 8.52 \times 10^{-3}, \\ \Delta y^{\text{WPD}} &= 5.46 \times 10^{-3}, \end{aligned} \quad (38)$$

again referring to Ref. [2] for the numerical values of the universal fermionic and bosonic parts in the parameters $\Delta x, \Delta y, \varepsilon$.

We turn to the standard values for the process-specific corrections, δ_f and Δy_f ($f = \nu, u, c, d, s, b$). The parameters δ_f are simply related to δ and given by

$$\delta_f = (Q_f^2 - 1)\delta = (Q_f^2 - 1) \times 0.20 \times 10^{-3}. \quad (39)$$

For fermions f with light isospin partners, Δy_f is obviously constant,

$$\begin{aligned} \Delta y_\nu &= \frac{\alpha(M_Z^2)}{8\pi c_0^2} \left[16(3 - 6s_0^2 + 4s_0^4)C_1 + 8c_0^2(2 - s_0^2)^2 C_2 + 4c_0^2(5 - 2s_0^2) \log(c_0^2) \right. \\ & \quad \left. + 55 - 96s_0^2 + 52s_0^4 \right], \\ \Delta y_u &= \Delta y_c = \frac{\alpha(M_Z^2)}{24\pi c_0^2} \left[\frac{16}{9}(27 - 90s_0^2 + 76s_0^4)C_1 + 8c_0^2(2 - s_0^2)^2 C_2 + 4c_0^2(5 - 2s_0^2) \log(c_0^2) \right. \end{aligned}$$

$$\begin{aligned}
& +55 - 140s_0^2 + \frac{908}{9}s_0^4 \Big], \\
\Delta y_d = \Delta y_s = & \frac{\alpha(M_Z^2)}{12\pi c_0^2} \left[\frac{16}{9}(27 - 72s_0^2 + 52s_0^4)C_1 + 8c_0^2(2 - s_0^2)^2C_2 + 4c_0^2(5 - 2s_0^2)\log(c_0^2) \right. \\
& \left. + 55 - 118s_0^2 + \frac{664}{9}s_0^4 \right], \tag{40}
\end{aligned}$$

or numerically,

$$\begin{aligned}
\Delta y_\nu &= -3.05 \times 10^{-3}, \\
\Delta y_u &= -0.82 \times 10^{-3}, \\
\Delta y_d &= -1.82 \times 10^{-3}. \tag{41}
\end{aligned}$$

However, Δy_b gets the above-mentioned top-mass dependent contributions via virtual W exchange. Proceeding analogously to our presentation [2] of $\Delta x, \Delta y, \varepsilon$, we split Δy_b into a “dominant” (*dom*) and a “remainder” (*rem*) term,

$$\Delta y_b = \Delta y_b(\text{dom}) + \Delta y_b(\text{rem}). \tag{42}$$

$\Delta y_b(\text{dom})$ represents an asymptotic expansion including constant terms for a high top-quark mass, and $\Delta y_b(\text{rem})$ summarizes the remainder, which vanishes by definition for $m_t \rightarrow \infty$. For $\Delta y_b(\text{dom})$ we obtain

$$\begin{aligned}
\Delta y_b(\text{dom}) = & \frac{\alpha(M_Z^2)}{12\pi c_0^2} \left[\frac{3}{s_0^2}t + \frac{17 - 16s_0^2}{2s_0^2}\log(t) + \frac{16}{9}(27 - 72s_0^2 + 52s_0^4)C_1 + \frac{6c_0^2}{s_0^2}(2 - s_0^2)^2C_2 \right. \\
& + \frac{12c_0^6}{s_0^2}(3 - s_0^2)C_3 + \left(1 - \frac{1}{2s_0^2}\right)(33 - 44s_0^2 + 12s_0^4)f_1(c_0^2) \\
& \left. + \left(\frac{13}{2s_0^2} - 13 + 6s_0^2\right)\log(c_0^2) + \frac{25}{3s_0^2} + \frac{392}{9} - 119s_0^2 + \frac{680s_0^4}{9} \right], \tag{43}
\end{aligned}$$

where we have used the shorthand

$$t = \frac{m_t^2}{M_Z^2}. \tag{44}$$

Since the full analytic form of the remainder is not very illuminating, $\Delta y_b(\text{rem})$ is given in App. A. Here, we just give the asymptotic expansion of $\Delta y_b(\text{rem})$ up to $\mathcal{O}(t^{-2})$,

$$\begin{aligned}
\Delta y_b(\text{rem}) = & \frac{\alpha(M_Z^2)}{48\pi s_0^2 c_0^2} \left[(21 - 20s_0^2)(5 - 6s_0^2)\log\left(\frac{t}{c_0^2}\right) + (3 - 4s_0^2)(33 - 44s_0^2 + 12s_0^4)f_1(c_0^2) \right. \\
& \left. + \frac{2601}{20} - \frac{33259s_0^2}{90} + \frac{1004s_0^4}{3} - 96s_0^6 \right] \frac{1}{t} \\
& + \frac{\alpha(M_Z^2)}{240\pi s_0^2 c_0^2} \left[(637 - 2244s_0^2 + 2630s_0^4 - 1020s_0^6)\log\left(\frac{t}{c_0^2}\right) \right. \\
& + (3 - 4s_0^2)(1 - 2s_0^2)(101 - 128s_0^2 + 30s_0^4)f_1(c_0^2) \\
& \left. + \frac{42123}{70} - \frac{303763s_0^2}{126} + \frac{31574s_0^4}{9} - 2178s_0^6 + 480s_0^8 \right] \frac{1}{t^2} \\
& + \mathcal{O}\left(\frac{\log(t)}{t^3}\right). \tag{45}
\end{aligned}$$

m_t/GeV	$\Delta y_b^{1\text{-loop}}/10^{-3}$	$\Delta y_b^{\text{lead}}/10^{-3}$	$\Delta y_b^{(1)}/10^{-3}$	$\Delta y_b^{(2)}/10^{-3}$	$\Delta y_b^{(3)}/10^{-3}$
120	4.08	10.25	-7.44	-3.40	1.70
160	10.14	19.36	1.67	7.15	9.62
180	13.73	24.01	6.32	11.69	13.46
240	26.51	38.97	21.28	25.73	26.46

Table 1: Comparison of Δy_b with the leading approximation, Δy_b^{lead} , and its asymptotic expansions $\Delta y_b^{(k)}$ where terms of $\mathcal{O}(\log(t)/t^k)$ are neglected.

Combining the results of (43) and (45), the asymptotic expansion of Δy_b reads numerically

$$\begin{aligned} \Delta y_b = & \left(3.47t + 7.70 \log(t) - 17.69 \right. \\ & + 17.13 \log(t)/t - 2.40/t + 14.26 \log(t)/t^2 + 7.45/t^2 \\ & \left. + \mathcal{O}(\log(t)/t^3) \right) \times 10^{-3}. \end{aligned} \quad (46)$$

In Tab. 1, we compare the exact values of Δy_b with the leading approximation Δy_b^{lead} and its asymptotic expansions $\Delta y_b^{(k)}$ where terms of $\mathcal{O}(\log(t)/t^k)$ are neglected. In particular, Δy_b^{lead} contains only the t - and $\log(t)$ -terms of (43), $\Delta y_b^{(1)}$ is identical with $\Delta y_b(\text{dom})$, and the numerical values of $\Delta y_b^{(2)}$, $\Delta y_b^{(3)}$ are obtained from (45). We find that including only the t - and $\log(t)$ -terms of (43), i.e. Δy_b^{lead} , or adding in addition the constant term to obtain $\Delta y_b^{(1)}$, is not a sufficient approximation for Δy_b , as can be clearly seen in Tab. 1. Therefore, we explicitly give further subleading terms in (45).

For a vanishing top-quark mass, m_t , the parameter Δy_b of course coincides with Δy_d as given in (40).

4 Other important corrections

4.1 Leading two-loop top-corrections

The explicit expressions for the parameters Δx , Δy , ε , Δy_b , which have been given in Ref. [2] and Sect. 3, are valid in one-loop approximation, i.e. in $\mathcal{O}(\alpha)$. Although up to now no complete two-loop calculation for the Z^0 and μ^\pm decay widths exists, the leading two-loop top-corrections have already been presented in the literature. QCD corrections of order $\mathcal{O}(\alpha_s \alpha t)$ were given in Ref. [18] and Ref. [19] for the ρ -parameter and Γ_b , respectively. Moreover, corrections of order $\mathcal{O}(\alpha^2 t^2)$ to the ρ -parameter for arbitrary Higgs mass were presented in Refs. [23, 24] and to Γ_b in Ref. [24]. It turns out that these leading heavy-top contributions can be naturally embedded into the parameters Δx , Δy , ε , Δy_b . The

m_t/GeV	$\Delta x^{1\text{-loop}}/10^{-3}$			$\Delta x _{\text{top},2\text{l}}^{\text{QCD}}/10^{-3}$	$\Delta x _{\text{top},2\text{l}}^{\text{weak}}/10^{-3}$		
	$M_H/\text{GeV} = 100, 300, 1000$				$M_H/\text{GeV} = 100, 300, 1000$		
120	7.49	6.71	5.44	−0.48	−0.02	−0.05	−0.06
160	11.98	11.20	9.93	−0.86	−0.05	−0.13	−0.19
180	14.48	13.70	12.43	−1.09	−0.07	−0.20	−0.30
240	23.22	22.45	21.17	−1.94	−0.13	−0.52	−0.92

m_t/GeV	$\Delta y_b^{1\text{-loop}}/10^{-3}$	$\Delta y_b _{\text{top},2\text{l}}^{\text{QCD}}/10^{-3}$	$\Delta y_b _{\text{top},2\text{l}}^{\text{weak}}/10^{-3}$		
			$M_H/\text{GeV} = 100, 300, 1000$		
120	4.08	-0.52	0.04	0.04	0.09
160	10.14	-0.92	0.15	0.12	0.22
180	13.73	-1.16	0.24	0.19	0.33
240	26.51	-2.07	0.83	0.62	0.86

Table 2: Leading two-loop top-corrections to Δx and Δy_b in comparison with one-loop results. “QCD” corresponds to the terms of order $\mathcal{O}(\alpha_s \alpha t)$, “weak” to the ones of order $\mathcal{O}(\alpha^2 t^2)$.

ρ -parameter enters merely Δx , and the process-specific heavy-top corrections to Γ_b yield only contributions to Δy_b , while Δy and ε are not affected. Following Ref. [24], we define

$$x_t \equiv \frac{\sqrt{2}G_\mu m_t^2}{16\pi^2} = \frac{\alpha(M_Z^2)}{16\pi s_0^2 c_0^2} t. \quad (47)$$

For the higher-order top-effects on Δx and Δy_b we finally obtain

$$\Delta x|_{\text{top},2\text{l}} = 9x_t^2 + 3x_t^2 \rho^{(2)}(m_t/M_H) + 3x_t \delta^{\text{QCD}}, \quad (48)$$

$$\Delta y_b|_{\text{top},2\text{l}} = 12x_t^2 + 4x_t^2 \tau_b^{(2)}(m_t/M_H) + 4x_t \delta_b^{\text{QCD}}, \quad (49)$$

where the first x_t^2 terms on the r.h.s. represent reducible (“squared one-loop”) contributions. The functions $\rho^{(2)}$ and $\tau_b^{(2)}$, which depend on the ratio m_t/M_H , can be explicitly found in Ref. [24]. The QCD parts simply read from Ref. [18]

$$\delta^{\text{QCD}} = -\frac{2\pi^2 + 6}{9} \frac{\alpha_s}{\pi} = -2.860 \frac{\alpha_s}{\pi} \quad (50)$$

and Ref. [19]

$$\delta_b^{\text{QCD}} = -\frac{\pi^2 - 3}{3} \frac{\alpha_s}{\pi} = -2.290 \frac{\alpha_s}{\pi}. \quad (51)$$

Both QCD contributions lead to a *screening* of the $\mathcal{O}(\alpha t)$ correction at one loop. In order to illustrate the influence of these leading two-loop effects, we compare $\Delta x|_{\text{top},2\text{l}}$

and $\Delta y_b|_{\text{top},2l}$ in Tab. 2 with the one-loop results for Δx and Δy_b for various Higgs and top-quark masses, respectively. We note that the experimental accuracy in Δx and Δy_b is of the order 5×10^{-3} (compare Sect. 5), which has to be compared with the order 1×10^{-3} and 0.3×10^{-3} for the $\mathcal{O}(\alpha_s \alpha t)$ and $\mathcal{O}(\alpha^2 t^2)$ terms (for $m_t \approx 175 \text{ GeV}$), respectively. Consequently, the (weak) $\mathcal{O}(\alpha^2 t^2)$ correction turns out to be negligible.

4.2 Finite-mass corrections

In the results of the previous sections all fermions except for the top-quark have been assumed to be massless. While this approximation is obviously excellent for the leptons and the quarks of the first and second fermion generation, the finite-mass effects of the b-quark can reach the order of the loop corrections even at the Z^0 -mass scale. Consequently, we include the $\mathcal{O}(m_b^2/M_Z^2)$ correction to $Z^0 \rightarrow b\bar{b}$, which is simply given by

$$\delta\Gamma_b|_{\text{mass}} = -\frac{\alpha(M_Z^2)}{8s_0^2c_0^2} N_{C,b} M_Z \frac{m_b^2}{M_Z^2}. \quad (52)$$

Terms of order $\mathcal{O}(m_b^4/M_Z^4)$ are completely negligible with respect to the experimental and theoretical uncertainties.

4.3 Higher-order QCD corrections

The QCD corrections to the hadronic decays of the Z^0 boson, $Z^0 \rightarrow q\bar{q}$, have been frequently discussed in the literature (see e.g. Refs. [13, 18, 19, 20, 21, 22]). Owing to finite-mass effects they are different for the vector and axial-vector parts $\Gamma_{V,q}$ and $\Gamma_{A,q}$, respectively. They are given by

$$\begin{aligned} \delta\Gamma_q|_{\text{QCD}} = & \Gamma_{V,q} \left\{ 12 \frac{\alpha_s}{\pi} \frac{m_q^2}{M_Z^2} \right\} \\ & + \Gamma_{A,q} \left\{ -6 \frac{\alpha_s}{\pi} \frac{m_q^2}{M_Z^2} \left[1 + 2 \log \left(\frac{m_q^2}{M_Z^2} \right) \right] \pm \frac{1}{3} \left(\frac{\alpha_s}{\pi} \right)^2 I \left(\frac{1}{4t} \right) \right\}, \end{aligned} \quad (53)$$

where the “ \pm ” refers to u/d-type quarks, respectively, and

$$\Gamma_{V,q} = \frac{\alpha(M_Z^2)}{16s_0^2c_0^2} M_Z (1 - 4s_0^2|Q_q|)^2, \quad \Gamma_{A,q} = \frac{\alpha(M_Z^2)}{16s_0^2c_0^2} M_Z. \quad (54)$$

The m_q -dependent corrections in (53) are only relevant for $q=b$ and have been taken from Ref. [20, 21]. The full analytical expression for the function $I(x)$ was presented in Ref. [22]. Instead, we use the excellent approximation

$$I \left(\frac{1}{4t} \right) = \frac{37}{4} + 3 \log(t) - 0.26 t^{-1} - 0.04 t^{-2} + \mathcal{O}(t^{-3}), \quad (55)$$

which can also be found there.

5 Analysis of the experimental data.

5.1 Input data.

For our analysis we use the experimental data presented at the Glasgow Conference [4],

$$\begin{aligned}
M_Z &= 91.1888 \pm 0.0044 \text{ GeV}, \\
\Gamma_T &= 2497.4 \pm 3.8 \text{ MeV}, \\
R &= \Gamma_h/\Gamma_l = 20.795 \pm 0.040, \\
\sigma_h &= \frac{12\pi\Gamma_l\Gamma_h}{M_Z^2\Gamma_T^2} = 41.49 \pm 0.12 \text{ nb}.
\end{aligned} \tag{56}$$

We take into account the correlation matrix for Γ_T , R , and σ_h ,

	Γ_T	R	σ_h
Γ_T	1.00	0.01	-0.11
R	0.01	1.00	0.13
σ_h	-0.11	0.13	1.00

(57)

since the other correlations (to M_Z and A_{FB}^l) are negligible. From (56) and (57) one derives

$$\begin{aligned}
\Gamma_l &= 83.96 \pm 0.18 \text{ MeV}, \\
\Gamma_h &= 1746 \pm 4 \text{ MeV}.
\end{aligned} \tag{58}$$

From the measured value of

$$R_{\text{bh}} = \frac{\Gamma_b}{\Gamma_h} = 0.2202 \pm 0.0020, \tag{59}$$

one then obtains

$$\Gamma_b = 384.5 \pm 3.6 \text{ MeV}. \tag{60}$$

From all asymmetries (A_{FB}^l , A_{pol}^τ , A^e , A_{FB}^b , A_{FB}^c) measured at LEP one deduces

$$\bar{s}_W^2(\text{LEP}) = 0.2321 \pm 0.0004. \tag{61}$$

Upon including the SLD result on $A_{\text{LR}}(\text{SLD})$ one has

$$\bar{s}_W^2(\text{LEP} + \text{SLD}) = 0.2317 \pm 0.0004. \tag{62}$$

In Ref. [2], we have shown the numerical results using both values for \bar{s}_W^2 . Here, we restrict ourselves to the LEP result (61) for numerical evaluations. The results to be obtained upon including the SLD value for \bar{s}_W^2 can be essentially inferred from our previous analysis. Finally, we have

$$\frac{M_{W^\pm}}{M_Z}(\text{UA2} + \text{CDF}) = 0.8798 \pm 0.0020. \tag{63}$$

Concerning the input value of $\alpha(M_Z^2)$, there is an experimental uncertainty due to the data on $e^+e^- \rightarrow q\bar{q} \rightarrow \text{hadrons}$, in particular in the low-energy region which strongly

affects the value of the dispersion integral employed when calculating $\alpha(M_Z^2)$. The value [12] of

$$\alpha(M_Z^2)^{-1} = 128.87 \pm 0.12 \quad (64)$$

has been supplemented by two new evaluations recently, yielding [25]

$$\alpha(M_Z^2)^{-1} = 129.08 \pm 0.10; \quad (65)$$

and [26]

$$\alpha(M_Z^2)^{-1} = 129.01 \pm 0.06. \quad (66)$$

We also note that the above values are consistent with the results

$$\alpha(M_Z^2)^{-1} = \begin{cases} 128.90 \pm 0.06, & \text{Ref. [27],} \\ 128.96 \pm 0.03, & \text{Ref. [28],} \end{cases} \quad (67)$$

based on the experimental data on $e^+e^- \rightarrow \text{hadrons}$ in the resonance regions in addition to stronger theoretical assumptions. In our analysis we will use the value (64) of Ref. [12] and indicate what will happen if this value is changed by the given uncertainty.

In addition to $\alpha(M_Z^2)$, we will use

$$G_\mu = 1.16639(2) \cdot 10^{-5} \text{ GeV}^{-2}, \quad (68)$$

M_Z from (56), the strong coupling constant [29]

$$\alpha_s = 0.118 \pm 0.007, \quad (69)$$

and the corresponding “on-shell” mass [21] of the b-quark,

$$m_b = 4.5 \text{ GeV}, \quad (70)$$

as input parameters.

5.2 Determination of the parameters from experiment

In Sects. 2.1 and 2.2, we presented the explicit formulae for the observables \bar{s}_W^2 , M_{W^\pm}/M_Z , and Γ_f in terms of the effective parameters. In view of the experimental uncertainties of these observables it is completely sufficient to consider the contributions of the parameters Δx , Δy , ε , Δy_f , δ , δ_q to the observables in linear order only, rendering our investigation very transparent. Hence, we write

$$\begin{aligned} \bar{s}_W^2 &= s_0^2 [1 + \sigma_x \Delta x + \sigma_y \Delta y + \sigma_\varepsilon \varepsilon + \sigma_\delta \delta], \\ \frac{M_{W^\pm}}{M_Z} &= c_0 [1 + \mu_x \Delta x + \mu_y \Delta y + \mu_\varepsilon \varepsilon + \mu_\delta \delta], \\ \Gamma_1 &= \Gamma_1^{(0)} [1 + \gamma_x^1 \Delta x + \gamma_y^1 \Delta y + \gamma_\varepsilon^1 \varepsilon + \gamma_\delta^1 \delta], \\ \Gamma_\nu &= \Gamma_\nu^{(0)} [1 + \gamma_x^\nu \Delta x + \gamma_y^\nu \Delta y + \gamma_{y_\nu}^\nu \Delta y_\nu], \\ \Gamma_q &= \Gamma_q^{(0)} [1 + \gamma_x^q \Delta x + \gamma_y^q \Delta y + \gamma_\varepsilon^q \varepsilon + \gamma_{y_q}^q \Delta y_q + \gamma_\delta^q \delta + \gamma_{\delta_q}^q \delta_q] + \delta\Gamma_q|_{\text{mass}} + \delta\Gamma_q|_{\text{QCD}}, \end{aligned} \quad (71)$$

where the coefficients σ_i , μ_i , γ_i^f can be easily read from (17), (28), (29), and the lowest-order contributions s_0^2 , c_0 , $\Gamma_f^{(0)}$ are defined in (15), (18), (30). Here, $\delta\Gamma_q|_{\text{mass}}$ and $\delta\Gamma_q|_{\text{QCD}}$ denote the finite-mass and higher-order QCD corrections given in (52) and (53), respectively. Based on (71), the hadronic width, Γ_h , and the total width, Γ_T , take the form

$$\begin{aligned}\Gamma_h &= 2\Gamma_u + 2\Gamma_d + \Gamma_b \\ &= \Gamma_h^{(0)} \left[1 + \gamma_x^h \Delta x + \gamma_y^h \Delta y + \gamma_\varepsilon^h \varepsilon + \gamma_{y_b}^h \Delta y_b + \gamma_{y_h}^h \Delta y_h + \gamma_\delta^h \delta_h \right] + \delta\Gamma_b|_{\text{mass}} + \delta\Gamma_h|_{\text{QCD}}, \\ \Gamma_T &= 3\Gamma_l + 3\Gamma_\nu + 2\Gamma_u + 2\Gamma_d + \Gamma_b \\ &= \Gamma_T^{(0)} \left[1 + \gamma_x^T \Delta x + \gamma_y^T \Delta y + \gamma_\varepsilon^T \varepsilon + \gamma_{y_b}^T \Delta y_b + \gamma_{y_h}^T \Delta y_h + \gamma_\delta^T \delta_T \right] + \delta\Gamma_b|_{\text{mass}} + \delta\Gamma_h|_{\text{QCD}}.\end{aligned}\tag{72}$$

Since the partial decay widths $Z^0 \rightarrow u\bar{u}, d\bar{d}$ cannot be measured separately, the parameters Δy_u and Δy_d cannot be resolved by experiment. They appear only in the combination

$$\Delta y_h = \frac{1}{2}(\Delta y_u + \Delta y_d) + \frac{s_0^2}{6c_0^2}(\Delta y_d - \Delta y_u)\tag{73}$$

in the hadronic and total widths in (72). The parameters δ_h and δ_T summarize the contributions of δ and δ_q .

The linearized equations (71) and (72) may now be used to extract the parameters Δx etc. from the experimental data on \bar{s}_W^2 etc. Conversely, employing the formulae for the standard values of the effective parameters Δx etc. presented in the previous sections and Ref. [2], the linearized equations (71) and (72) can be used to evaluate the complete standard model prediction for the observables at one loop. In order to obtain also the leading two-loop top-corrections in Δx and Δy_b (see Sect. 4.1) correctly, (71) and (72) have to be completed by the corresponding quadratic terms.

In our analysis of the experimental data, we employ a *two-step procedure*. In a *first step*, we determine the experimental values of the six parameters

$$\Delta x, \Delta y, \varepsilon; \quad \Delta y_b, \Delta y_h, \Delta y_\nu\tag{74}$$

from the six experimental data

$$\bar{s}_W^2, M_{W^\pm}/M_Z, \Gamma_l; \quad \Gamma_h, \Gamma_T, \Gamma_b,\tag{75}$$

where the contributions of δ , δ_q , $\delta\Gamma_b|_{\text{mass}}$, and $\delta\Gamma_h|_{\text{QCD}}$ are taken from theory.⁴ The results for $\Delta x, \Delta y, \varepsilon$ only depend on the input for $\Gamma_l, \bar{s}_W^2, M_{W^\pm}/M_Z$. They are shown in Figs. 4a)-6a). A comparison of these 83 % C.L. contours in the respective planes with our previous ones [2] shows a decrease in the sizes of the ellipses due to a somewhat decreased experimental error. The main conclusion from these results has been presented in Refs. [1, 2]:

⁴Note that $\delta\Gamma_h|_{\text{QCD}}$ according to (53) (smoothly) depends on the top mass, m_t , in order $\mathcal{O}(\alpha_s^2)$. For the evaluations, we choose $m_t = 175 \text{ GeV}$ and remark that e.g. a change in m_t of $\pm 50 \text{ GeV}$ leads to a variation of at most 0.6×10^{-3} in Δy_b^{exp} , which is very much smaller than the experimental error.

the data have reached such a high accuracy *that contributions beyond the fermion-loop contributions to the γ , W^\pm and Z^0 propagators are clearly required*. In particular, the data require a significant contribution to Δy which in the standard electroweak theory is provided by vertex corrections to the $W^\pm ff'$ and $Z^0 ff$ vertices which involve trilinear vector-boson self-couplings. On the other hand, Δy is practically independent [5] of the mass of the Higgs scalar and the concept of the Higgs mechanism. The experimental data on ε and Δx are well approximated by

$$\Delta x \approx \Delta x^{\text{ferm}}, \quad \varepsilon \approx \varepsilon^{\text{ferm}}. \quad (76)$$

The numerical values for the parameters Δx^{exp} , Δy^{exp} , ε^{exp} , shown in Figs. 4a)-6a), are given by

$$\begin{aligned} \Delta x^{\text{exp}} &= (9.8 \pm 4.7 \pm 0.2 \pm 0) \times 10^{-3}, \\ \Delta y^{\text{exp}} &= (4.6 \pm 4.9 \pm 0.2 \pm 0) \times 10^{-3}, \\ \varepsilon^{\text{exp}} &= (-6.1 \pm 2.0 \mp 0.7 \pm 0) \times 10^{-3}, \end{aligned} \quad (77)$$

where the first error is statistical (1σ), the second due to the deviation by replacing $\alpha(M_Z^2)^{-1} \rightarrow \alpha(M_Z^2)^{-1} \pm \delta\alpha(M_Z^2)^{-1}$ according to (64), and the third due to $\alpha_s \rightarrow \alpha_s \pm \delta\alpha_s$ according to (69). In Figs. 4-6, the shift of the (center of the) ellipses as a result of the changes $\alpha(M_Z^2)^{-1} \rightarrow \alpha(M_Z^2)^{-1} + \delta\alpha(M_Z^2)^{-1}$ and $\alpha_s \rightarrow \alpha_s + \delta\alpha_s$ is indicated by an arrow in the upper left-hand corner. Note that the uncertainties in Δx^{exp} etc. induced by $\delta\alpha(M_Z^2)^{-1}$ and $\delta\alpha_s$ are mainly due to the subtractions of the “lowest-order” contributions, s_0^2 , c_0 , $\Gamma_1^{(0)}$ etc., and the mass and QCD corrections, $\delta\Gamma_q|_{\text{mass}}$ and $\delta\Gamma_q|_{\text{QCD}}$, in (71) and (72), respectively. The theoretical predictions, Δx^{th} etc., are only influenced via higher orders and lead to entirely negligible shifts in the figures.

In Figs. 7a)-9a), we show the results for Δy_b in conjunction with $\Delta x, \Delta y, \varepsilon$. As expected from the known discrepancy between experiment and theory in Γ_b , the data clearly indicate a value of Δy_b which does not show the expected enhancement due to a large mass of the top-quark. Numerically, Δy_b^{exp} is given by

$$\Delta y_b^{\text{exp}} = (-9.5 \pm 8.2 \pm 0.0 \pm 1.8) \times 10^{-3}. \quad (78)$$

In Figs. 4-9, we also show the results of taking into account fermion loops only, obviously corresponding to $\Delta y_b = 0$ as Δy_b gets no fermion-loop contributions. We have also indicated for comparison the value of Δy_d in Figs. 7-9 by an arrow, which corresponds to Δy_b for $m_t \rightarrow 0$. It seems that the data on Γ_b prefer a theoretical value for Δy_b which does not contain the effect of the m_t -dependent vertex correction.

For the remaining parameters, Δy_h and Δy_ν , which are not shown in figures, we find

$$\begin{aligned} \Delta y_h^{\text{exp}} &= (1.2 \pm 2.8 \pm 0.0 \pm 1.9) \times 10^{-3}, \\ \Delta y_\nu^{\text{exp}} &= (1.3 \pm 7.7 \pm 0.0 \pm 0.0) \times 10^{-3}. \end{aligned} \quad (79)$$

These values are seen to be consistent with the theoretical predictions

$$\begin{aligned} \Delta y_h^{\text{th}} &= -1.37 \times 10^{-3}, \\ \Delta y_\nu^{\text{th}} &= -3.05 \times 10^{-3}. \end{aligned} \quad (80)$$

We note that the uncertainty in $\alpha(M_Z^2)$ strongly influences the parameter ε and accordingly the determination of the Higgs mass, while it is fairly irrelevant for the remaining parameters. The uncertainty in α_s is reflected in the determination of the (hadronic) parameters Δy_b and Δy_h .

We turn to the *second step* of our analysis. As noted in Sect. 3, the theoretical predictions for Δy_h and Δy_ν are actually on the same footing as the theoretical predictions for the fermion-loop contributions to the γ, W^\pm and Z^0 propagators. Both predictions involve vector-boson fermion couplings only, and are (consequently) independent of the (empirically unknown) vector-boson self-couplings. Motivated by the consistency between theory and experiment for $\Delta y_h, \Delta y_\nu$ in (80) and (79), we now impose the assumption that the process-specific vertex corrections $\Delta y_h, \Delta y_\nu$ are given by the standard values (80). Accordingly, the number of six free (fit) parameters, thus, being reduced to four, $\Delta x, \Delta y, \varepsilon$ and Δy_b . Concerning the experimental input, we will discriminate between six ($\Gamma_l, \bar{s}_W, M_{W^\pm}/M_Z, \Gamma_h, \Gamma_T, \Gamma_b$) and five ($\Gamma_l, \bar{s}_W, M_{W^\pm}/M_Z, \Gamma_h, \Gamma_T$) input data. This discrimination allows us to analyze the influence of the data for Γ_b on the results for $\Delta x, \Delta y, \varepsilon$, and Δy_b . We refer to these cases by “with Γ_b ” and “without Γ_b ”, respectively.

We find

$$\begin{aligned} \text{with } \Gamma_b : \quad \Delta x^{\text{exp}} &= (9.6 \pm 4.7 \pm 0.2 \mp 0.0) \times 10^{-3}, \\ \Delta y^{\text{exp}} &= (5.6 \pm 4.8 \pm 0.2 \pm 0.4) \times 10^{-3}, \\ \varepsilon^{\text{exp}} &= (-5.2 \pm 1.8 \mp 0.7 \pm 0.3) \times 10^{-3}, \\ \Delta y_b^{\text{exp}} &= (-3.3 \pm 5.9 \pm 0.0 \pm 5.8) \times 10^{-3}. \end{aligned} \quad (81)$$

A comparison of these numerical results⁵ (81) with the ones of the six-parameter analysis (77), (78) does not reveal dramatic differences in $\Delta x, \Delta y, \varepsilon$. The experimental result for Δy_b is shifted into the direction of the theoretical prediction by roughly one standard deviation. This is also evident from comparing Figs. 4a)-9a) with Figs. 4b)-9b). The sizes of the 83% C.L. ellipses, as expected, are slightly decreased in Figs. 4b)-9b) due to the smaller number of four (fit) parameters. On the other hand, the dependence on α_s is somewhat stronger in the four-parameter fit. As in Figs. 4a)-9a) for the six-parameter

⁵Note added: While proofreading the final version of the present paper, we obtained the final draft of the preprint CERN-TH 7536/94 by G. Altarelli, R. Barbieri, and F. Caravaglios, which contains an update of their analysis of the Z^0 data in terms of the parameters $\varepsilon_{1,2,3}, \varepsilon_b$ [9]. These parameters are related to ours by

$$\begin{aligned} \varepsilon_1 &= \Delta x - \Delta y + 4s_0^2\delta = \Delta x - \Delta y + 0.2 \times 10^{-3}, \\ \varepsilon_2 &= -\Delta y + 2s_0^2\delta = -\Delta y + 0.1 \times 10^{-3}, \\ \varepsilon_3 &= -\varepsilon + \delta = -\varepsilon + 0.2 \times 10^{-3}, \\ \varepsilon_b &= -\Delta y_b/2 + 4s_0^2[(2s_0^2 - 3)\delta + s_0^2\delta_b]/(9 - 6s_0^2) = -\Delta y_b/2 - 0.1 \times 10^{-3}. \end{aligned}$$

Repeating our analysis for precisely the same experimental data used there ($\bar{s}_W^2(\text{LEP} + \text{SLD}) = 0.2317 \pm 0.0004$, $R_{bh} = 0.2192 \pm 0.0018$), we reproduced the values for $\varepsilon_{1,2,3}$, within 0.1×10^{-3} and the one for ε_b within 1×10^{-3} . In view of the differences between the two analyses (lowest-order and QCD subtractions etc.), a deviation in ε_b of this order is to be expected. We note that the dominance of the fermion loops is clearly visible in our set of parameters, where $\Delta x \approx \Delta x^{\text{ferm}}$ in the standard electroweak theory, while this fact is concealed in the linear combination, ε_1 .

analysis, in Figs. 4b)-9b) we indicate the $\alpha(M_Z^2)$ - and α_s -uncertainties of the experimental ellipses by arrows in the upper left-hand corner. The long arrow for $\delta\alpha_s$ in Figs. 7b)-9b) corresponds to the case “without Γ_b ”, where Γ_b is excluded from the fit. Here, we find

$$\begin{aligned} \text{without } \Gamma_b : \quad & \Delta x^{\text{exp}} = (9.6 \pm 4.7 \pm 0.2 \pm 0.0) \times 10^{-3}, \\ & \Delta y^{\text{exp}} = (5.0 \pm 4.8 \pm 0.2 \pm 0.0) \times 10^{-3}, \\ & \varepsilon^{\text{exp}} = (-5.7 \pm 1.8 \mp 0.7 \pm 0.0) \times 10^{-3}, \\ & \Delta y_b^{\text{exp}} = (1.0 \pm 7.2 \pm 0.0 \pm 8.8) \times 10^{-3}. \end{aligned} \quad (82)$$

As expected, the main change in (82) relative to (81) occurs in Δy_b , which is moved upwards in the direction of better agreement with the theoretical expectation for a heavy top-quark. We note that even upon excluding the data for Γ_b , the agreement of the fit with theory is not perfect. If we impose the additional constraint of $m_t \approx 175$ GeV, the value of m_t indicated by the direct searches [30], the theoretical prediction is at the edge of the experimentally allowed region, rather than in the center. Inspecting the α_s -dependence of the results, we see that the agreement between experiment and theory for $m_t \approx 175$ GeV will be considerably improved if α_s approaches higher values such as $\alpha_s + \delta\alpha_s$. This conclusion is in accordance with α_s -fits performed by other groups.

6 Conclusions

Upon having included the hadronic Z^0 decays in our analysis, we reemphasize our previous conclusions. The parameters Δx , Δy , ε , and Δy_b , introduced as a parametrization of $SU(2)$ -symmetry breaking in an effective Lagrangian, are empirically found to have a magnitude typical for radiative corrections. This in itself is a major triumph of the $SU(2)$ symmetry properties embodied in the standard electroweak theory. Moreover, discriminating between the pure fermion-loop corrections (only dependent on the couplings of the fermions to the vector bosons) and the remaining “bosonic” corrections (dependent on the non-Abelian vector-boson couplings and the Higgs scalar), we have found that the experiments have become sufficiently accurate to require corrections beyond pure fermionic vacuum-polarization effects. More specifically, the data require a non-vanishing value of the bosonic contribution to Δy which in the standard model is induced by (combined) vertex corrections at the $W^+l\bar{\nu}$ ($W^-\nu l$) and $Z^0 l\bar{l}$ vertices. These corrections depend on the non-Abelian structure of the vector-boson sector of the theory.

The leading two-loop top-corrections to the ρ -parameter and $Z^0 \rightarrow b\bar{b}$ are naturally absorbed by Δx and Δy_b , respectively. For a top mass of about 175 GeV, the $\mathcal{O}(\alpha_s\alpha t)$ terms are of the order 1×10^{-3} , the $\mathcal{O}(\alpha^2 t^2)$ terms of the order 0.3×10^{-3} , which have both to be confronted with the present experimental error of the order of 5×10^{-3} in Δx and Δy_b .

Technically, in order to encourage experimentalists to carry out such an analysis themselves with future precision data, we have given all relevant analytic formulae explicitly which are necessary to extract the parameters Δx , Δy etc. from the data by a fitting procedure. Moreover, the standard-model values for these parameters have been explicitly displayed in an analytically compact form, which can also be easily evaluated numerically.

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Appendix

A Remainder of Δy_b

In Sect. 3, we have split the parameter Δy_b into a dominant and remainder part $\Delta y_b(dom)$ and $\Delta y_b(rem)$, respectively, but we have only given the first few asymptotic terms of the remainder. Recall that $\Delta y_b(rem)$ is defined such that $\Delta y_b(dom)$ contains all contributions which do not vanish for $t \rightarrow \infty$. Here, we present the full formula,

$$\begin{aligned} \Delta y_b(rem) = & \frac{\alpha(M_Z^2)}{24\pi s_0^2 c_0^2} \left[-\frac{23}{3} + \frac{71s_0^2}{9} + 8s_0^4 c_0^2 + \frac{3c_0^4(3-2s_0^2)}{t-c_0^2} - \left(\frac{33}{2} - 21s_0^2 + 4s_0^4 \right) t \right. \\ & - 2(3-2s_0^2)t^2 - (17-16s_0^2) \log\left(\frac{t}{c_0^2}\right) \\ & + \frac{\log(t/c_0^2)}{(t-c_0^2)^2} \left\{ 2c_0^6(3-4s_0^2)(5-2s_0^2) - 4c_0^4(27-41s_0^2+12s_0^4)t \right. \\ & + 2c_0^2(45-67s_0^2+18s_0^4)t^2 - (15-10s_0^2-8s_0^4)t^3 - 6(1-2s_0^2)t^4 \} \\ & + \left\{ -30 + 82s_0^2 - 68s_0^4 + 16s_0^6 + 4(3-6s_0^2+2s_0^4)t - 8s_0^2t^2 \right\} f_1(t) \\ & + \left\{ 33 - 110s_0^2 + 100s_0^4 - 24s_0^6 - 3(7-12s_0^2+4s_0^4)t \right. \\ & \left. - 6(1-2s_0^2)t^2 \right\} f_1(c_0^2) \\ & + 2 \left\{ -2c_0^2(3-4s_0^2)(2-s_0^2)^2 + 4c_0^2(6-8s_0^2+3s_0^4)t - (9-10s_0^2)t^2 \right. \\ & \left. + 4s_0^2t^3 \right\} C_4(t) \\ & + 6 \left\{ -4c_0^6(3-s_0^2) + c_0^2(7-3s_0^2)(1-2s_0^2)t - (3-4s_0^2)t^2 \right. \\ & \left. - (1-2s_0^2)t^3 \right\} C_5(t) \Big]. \end{aligned} \quad (83)$$

The auxiliary functions $f_1(x)$, $C_4(t)$, $C_5(t)$ are explicitly given in App. B.

B Auxiliary functions

Here, we list the explicit expressions for the auxiliary functions which have been used in Sect. 3 and App. A. $f_1(x)$ is defined by

$$f_1(x) = \text{Re} \left[\beta_x \log \left(\frac{\beta_x - 1}{\beta_x + 1} \right) \right], \quad \text{with } \beta_x = \sqrt{1 - 4x + i\epsilon}. \quad (84)$$

The constants C_1, C_2, C_3 and the functions $C_4(t), C_5(t)$ are shorthands for the scalar three-point integrals occuring in the process dependent parts of the decay $Z^0 \rightarrow f\bar{f}$,

$$\begin{aligned}
C_1 &= M_Z^2 \operatorname{Re} [C_0(0, 0, M_Z^2, 0, M_Z, 0)] = -\frac{\pi^2}{12} = -0.8225, \\
C_2 &= M_Z^2 \operatorname{Re} [C_0(0, 0, M_Z^2, 0, M_W, 0)] = \frac{\pi^2}{6} - \operatorname{Re} \left[\operatorname{Li}_2 \left(1 + \frac{1}{c_0^2} \right) \right] = -0.8037, \\
C_3 &= M_Z^2 \operatorname{Re} [C_0(0, 0, M_Z^2, M_W, 0, M_W)] = \operatorname{Re} \left[\log^2 \left(\frac{i\sqrt{4c_0^2 - 1} - 1}{i\sqrt{4c_0^2 - 1} + 1} \right) \right] = -1.473, \\
C_4(t) &= M_Z^2 \operatorname{Re} [C_0(0, 0, M_Z^2, m_t, M_W, m_t)], \\
C_5(t) &= M_Z^2 \operatorname{Re} [C_0(0, 0, M_Z^2, M_W, m_t, M_W)]. \tag{85}
\end{aligned}$$

The first three arguments of the C_0 -function label the external momenta squared, the last three the inner masses of the corresponding vertex diagram. All C_0 -functions occuring in (85) follow from the more general result

$$\begin{aligned}
C_0(0, 0, s, m_1, m_0, m_1) &= -\frac{1}{s} \left[\operatorname{Li}_2 \left(1 - \frac{m_0^2}{m_1^2} \right) - \operatorname{Li}_2 \left(1 + \frac{m_0^2}{m_1^2} x_2 \right) - \operatorname{Li}_2(1 + x_2) \right. \\
&\quad \left. + \frac{1}{2} \log^2(-x_1) + \sum_{\sigma=\pm 1} \{ \operatorname{Li}_2(1 - x_1^\sigma x_2) + \eta(-x_2, -x_1^\sigma) \log(1 - x_1^\sigma x_2) \} \right], \tag{86}
\end{aligned}$$

with

$$x_1 = \frac{1 + \beta}{1 - \beta}, \quad x_2 = \frac{s + i\epsilon - m_1^2 + m_0^2}{m_1^2 - m_0^2}, \quad \beta = \sqrt{1 - \frac{4m_1^2}{s + i\epsilon}}. \tag{87}$$

The dilogarithm $\operatorname{Li}_2(x)$ and the η -function $\eta(x, y)$ are defined as usual,

$$\operatorname{Li}_2(x) = -\int_0^x \frac{dt}{t} \log(1 - t), \quad -\pi < \arccos(1 - x) < \pi, \tag{88}$$

$$\eta(x, y) = \log(xy) - \log(x) - \log(y), \quad -\pi < \arccos(x), \arccos(y) < \pi. \tag{89}$$

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Fig. a)

Fig. b)

Figure 4: 83% C.L. ellipse in the Δx - ε -plane obtained in a) a six-parameter analysis $(\Delta x, \Delta y, \varepsilon, \Delta y_b, \Delta y_h, \Delta y_\nu)$, b) a four-parameter analysis $(\Delta x, \Delta y, \varepsilon, \Delta y_b)$ of the data. The full standard model predictions are shown for Higgs masses of 100 GeV (dotted with diamonds), 300 GeV (long-dashed-dotted), 1 TeV (short-dashed-dotted) parametrized by the top-quark mass ranging from 100-240 GeV in steps of 20 GeV. The pure fermion-loop prediction is also shown (short-dashed curve with squares) for the same top-quark masses.

Fig. a)

Fig. b)

Figure 5: 83% C.L. ellipse in the Δx - Δy -plane. See also caption of Fig. 4.

Fig. a)

Fig. b)

Figure 6: 83% C.L. ellipse in the ε - Δy -plane. See also caption of Fig. 4.

Fig. a)

Fig. b)

Figure 7: 83% C.L. ellipse in the Δy_{b} - Δx -plane. See also caption of Fig. 4. The short/long arrows for $\delta\alpha_s$ in Fig. b) correspond to the cases with/without Γ_{b} , respectively.

Fig. a)

Fig. b)

Figure 8: 83% C.L. ellipse in the Δy_b - Δy -plane. See also caption of Fig. 4. The short/long arrows for $\delta\alpha_s$ in Fig. b) correspond to the cases with/without Γ_b , respectively.

Fig. a)

Fig. b)

Figure 9: 83% C.L. ellipse in the Δy_b - ε -plane. See also caption of Fig. 4. The short/long arrows for $\delta\alpha_s$ in Fig. b) correspond to the cases with/without Γ_b , respectively.